Jack L. Wray,[1] M.F.S.; James E. McNeil, Jr.,[2] M.F.S.; and Walter F. Rowe,[3] Ph.D.

Comparison of Methods for Estimating Range of Fire Based on the Spread of Buckshot Patterns

REFERENCE: Wray, J. L., McNeil, J. E., Jr., and Rowe, W. F., **"Comparison of Methods for Estimating Range of Fire Based on the Spread of Buckshot Patterns,"** *Journal of Forensic Sciences*, JFSCA, Vol. 28, No. 4, Oct. 1983, pp. 846-857.

ABSTRACT: Three methods of measuring the spread of shotgun pellet patterns for the purpose of estimating the range of fire were applied to a series of 72 00 buckshot patterns test-fired at distances ranging from 3.6 to 10.7 m (12 to 35 ft). The methods applied were (1) the "effective shot dispersion" method of Mattoo and Nabar, (2) a method in which the area of the smallest circumscribed rectangle that will just enclose the pellet pattern is calculated, and (3) an overlay method for determining the radius of the smallest circumscribed circle that will just enclose the pellet pattern. Regression analysis was applied to the resulting measurements of the spread of the pellet patterns. The "effective shot dispersion" was found to give the best fit to a linear function and the best range-of-fire estimates. The area of the pellet patterns was found to be a quadratic function of the range of fire; this measure of pellet pattern spread was also found to have very large shot-to-shot variations. The square root of the area of the pellet pattern was found to be a linear function of the range and to give acceptable range-of-fire estimates.

KEYWORDS: criminalistics, ballistics, shotguns

While conducting a series of experiments on the ballistics of shotguns, we found that we needed a method for measuring the spread of the pellet patterns produced by 00 buckshot loads. Generally, forensic scientists are interested in determining the spread of a pellet pattern on a target (frequently a human being) so that the range of fire may be estimated. For 00 buckshot patterns, at least two methods for measuring the spread of the pellet pattern have been advocated: Mattoo and Nabar [*1*] proposed calculating the "effective shot dispersion" by a basically statistical method; other investigators [*2*] have used a method in which the area of the smallest circumscribed rectangle that just encloses the pellet pattern is calculated. The method of Mattoo and Nabar is cumbersome to apply because of the many measurements and calculations required, while the method using the area measure is quick and

The opinions or assertions contained herein are the private views of the authors and are not to be construed as official or as reflecting the views of the Department of the Air Force or the Department of Defense. Received for publication 31 July 1982; revised manuscript 2 March 1983; accepted for publication 3 March 1983.

[1]Graduate student, Department of Forensic Sciences, George Washington University, Washington, DC and formerly, Fellow in Forensic Medicine, Armed Forces Institute of Pathology, Washington, DC; presently, captain, U.S. Air Force.

[2]Graduate student, Department of Forensic Sciences, George Washington University, Washington, DC and formerly, Fellow in Forensic Medicine, Armed Forces Institute of Pathology, Washington, DC; presently, 1st lieutenant, U.S. Air Force.

[3]Associate professor, Department of Forensic Sciences, George Washington University, Washington, DC.

easy to use. Beyond matters of convenience, however, there seems to be little basis for preferring one method over the other. No studies have been published that compare these methods in terms of the statistical correlations between range of fire and the measurement of pellet pattern spread being used and in terms of the confidence limits to be placed on range-of-fire estimates obtained by each method. To remedy this, we have fired a series of 00 buckshot patterns at various ranges and applied to each pattern the method of Mattoo and Nabar, the method in which the area of the smallest circumscribed rectangle is used, and an overlay method for obtaining the radius of the smallest circle that will just enclose the pallet pattern. The measurements of pellet pattern spread obtained were then subjected to linear regression analysis so that we could determine which method showed the best correlation between its measurement of pattern spread and the range of fire and which method allowed the smallest confidence limits for range-of-fire estimates.

Experimental Procedure

A Remington Model 12, 12-gauge shotgun with a 508-mm (20-in.) cylinder-bored barrel was used to fire Remington 12-gauge, 70-mm (2¾-in.) 00 buckshot cartridges (nominal pellet diameter, 8 mm [0.33 in.]; nine pellets per round) at 914 by 914-mm (36 by 36-in.) butcher paper. All the cartridges belonged to a single batch. The shots were fired at ranges of 3.6 to 10.7 m (12 to 35 ft) in 0.3-m (1-ft) increments. Three shots were fired at each range, for a total of 72 rounds. No shots were fired within the 3.6-m (12-ft) range; within this range the buckshot did not make individual holes in the target material.

The spread of each pellet pattern was determined in three ways. A grid overlay (engineering Mylar® with 25-mm [1-in.] grids subdivided into 2.5-mm [1⁄10-in.] grids) was placed over the pellet pattern with its y axis oriented vertically. The x and y coordinates of each pellet hole were then recorded. The center of mass (com) of the pellet pattern was then calculated from the following formulas:

$$x_{com} = \sum_{i=1}^{9} x_i/9$$
$$y_{com} = \sum_{i=1}^{9} y_i/9 \qquad (1)$$

The dispersals S of the pattern were then determined by calculation of the second moment [*3*] of the pattern:

$$S = \left\{ \sum_{i=1}^{9} [(x_i - x_{com})^2 + (y_i - y_{com})^2]/9 \right\}^{1/2} \qquad (2)$$

This treatment is essentially the same as that suggested by Mattoo and Nabar [*1*]. The area A of the smallest circumscribed rectangle that would enclose the pellet pattern was obtained as a product of the largest difference in the x coordinates and the largest difference in the y coordinates.

A 203-mm (8-in.) plastic overlay marked with concentric circles (Fig. 1) was placed over each pellet pattern and moved about until its center appeared to coincide with the center of the pellet pattern. The radius R of the smallest circumscribed circle that would completely enclose the pellet pattern was then read directly from the overlay.

Regression Analysis

Two problems confront us as we examine the results of our test firings (Table 1 and Fig. 2). We must determine the appropriate functional relationships between S, R, and A and the

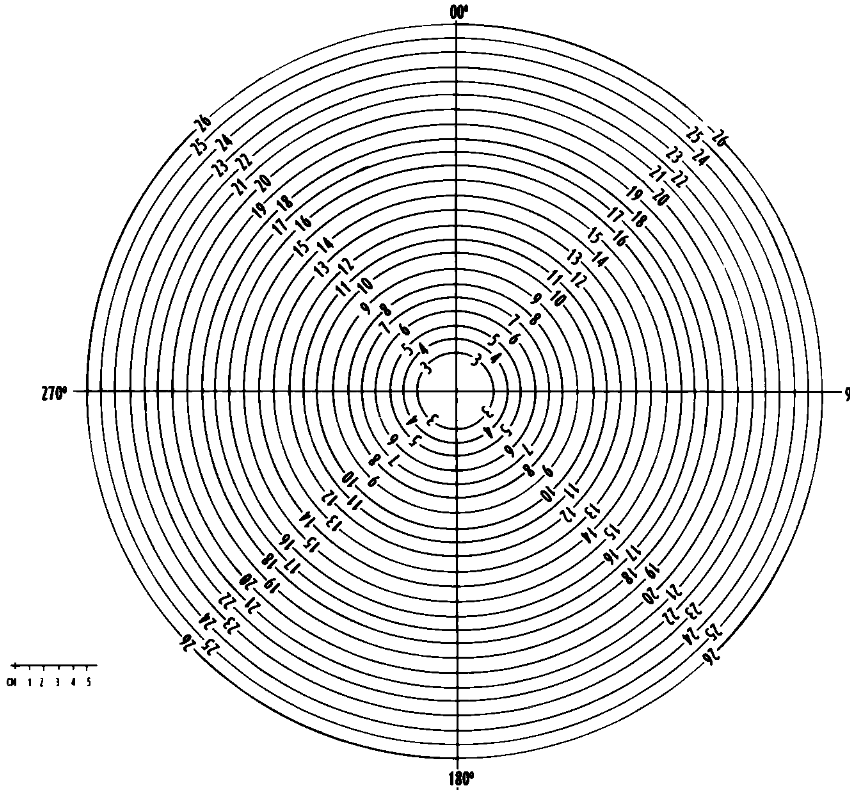

FIG. 1—*Plastic overlay used to determine radius of circle that will enclose pellet pattern.*

range of fire. This is the problem of establishing the proper regression line. Knowledge of the correct regression line is essential if we wish to estimate the range from which a questioned shotgun pellet pattern with known values of S, R, or A was fired. We must also estimate the degree to which the values of S, R, and A tend to scatter from their respective regression lines. Without such information no confidence interval for an estimated range can be obtained.

Examination of Fig. 2 reveals that S and R are apparently linear functions of the range, while A is not. Because A is approximately proportional to R^2, A would be expected to be a quadratic function of the range. On the other hand, $\sqrt{A}$ may be linear with respect to range. Figure 2 strongly suggests this to be the case.

The functional relationship between S, R, and $\sqrt{A}$ and the range of fire was chosen to be

$$y = a + bx \tag{3}$$

where x = range of fire; $y = S$, R, or $\sqrt{A}$; and a and b are the regression coefficients. When the standard deviations of the y's are constant, the method of least squares applied to n pairs of values (x_i, y_i) yields the following formulas for a and b [*4*, p. 104]:

$$a = 1/\Delta\left(\sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i y_i\right) \tag{4}$$

$$b = 1/\Delta\left(n \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i\right) \tag{5}$$

$$\Delta = n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2 \tag{6}$$

The linear correlation coefficient r is given by [4, p. 12]

$$r \equiv \frac{n \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\left\{\left[n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2\right]\left[n \Sigma y_i^2 - \left(\Sigma y_i\right)^2\right]\right\}^{1/2}} \tag{7}$$

The coefficient r is a commonly used estimator of the degree of the linear correlation between the x's and y's.

The scatter of the dependent variables from the regression line is estimated by the standard error of estimate S_e [4, p. 114] where

$$S_e = \left\{\frac{\sum_{i=1}^{n} (y_i - a - bx_i)^2}{n - 2}\right\}^{1/2} \tag{8}$$

The confidence interval for an estimated range $\hat{x}_o$ calculated from a y_o which is the mean of m y values is given by [3, p. 287]

$$\hat{x}_o \pm \frac{tS_e}{|b|} \left\{\frac{1}{m} + \frac{1}{n} + \left[\left(y_o - a - b\bar{x}\right)^2\right] \Big/ \left[b^2 \sum_{i=1}^{n} \left(x - \bar{x}\right)^2\right]\right\}^{1/2} \tag{9}$$

TABLE 1—*Experimental results.*[a]

Range, ft	S, in.	R, in.	A, in.^{2}	$\sqrt{A}$, in.
12	0.9 (0.1)[b]	1.1 (0.1)	3.0 (0.2)	1.7 (0.1)
13	0.9 (0.1)	1.1 (0.1)	3.6 (1.1)	1.9 (0.3)
14	0.9 (0.05)	1.1 (0.1)	3.4 (0.5)	1.8 (0.1)
15	1.0 (0.1)	1.3 (0.1)	3.9 (0.6)	2.0 (0.2)
16	1.0 (0.1)	1.3 (0.1)	4.0 (0.9)	2.0 (0.2)
17	1.1 (0.1)	1.3 (0.1)	5.2 (0.8)	2.3 (0.2)
18	1.1 (0.2)	1.4 (0.3)	5.6 (1.9)	2.3 (0.4)
19	1.2 (0.1)	1.4 (0.2)	6.5 (1.3)	2.7 (0.1)
20	1.3 (0.2)	1.7 (0.1)	7.3 (1.8)	2.7 (0.3)
21	1.2 (0.2)	1.6 (0.2)	6.9 (1.7)	2.6 (0.3)
22	1.4 (0.05)	1.7 (0.1)	8.0 (1.7)	2.8 (0.3)
23	1.5 (0.2)	1.9 (0.3)	9.5 (2.8)	3.1 (0.5)
24	1.3 (0.3)	1.7 (0.4)	7.9 (4.0)	2.7 (0.7)
25	1.5 (0.3)	1.9 (0.5)	11.6 (7.0)	3.3 (1.0)
26	1.6 (0.1)	2.0 (0.3)	11.6 (3.0)	3.4 (0.4)
27	1.9 (0.2)	2.6 (0.3)	16.1 (4.1)	4.0 (0.5)
28	1.8 (0.1)	2.2 (0.3)	14.3 (1.5)	3.8 (0.2)
29	2.0 (0.1)	2.5 (0.1)	15.6 (2.9)	3.9 (0.4)
30	2.0 (0.3)	2.6 (0.3)	17.6 (3.9)	4.2 (0.5)
31	2.1 (0.2)	2.7 (0.3)	18.8 (4.1)	4.3 (0.5)
32	2.0 (0.03)	2.7 (0.3)	17.0 (1.4)	4.1 (0.2)
33	2.2 (0.2)	2.9 (0.2)	22.1 (4.6)	4.7 (0.5)
34	2.1 (0.2)	2.6 (0.2)	19.7 (3.8)	4.4 (0.4)
35	2.4 (0.2)	3.1 (0.2)	26.4 (5.7)	5.1 (0.5)

[a] 1 in. = 25.4 mm and 1 ft = 0.3 m.
[b] Mean (standard deviation).

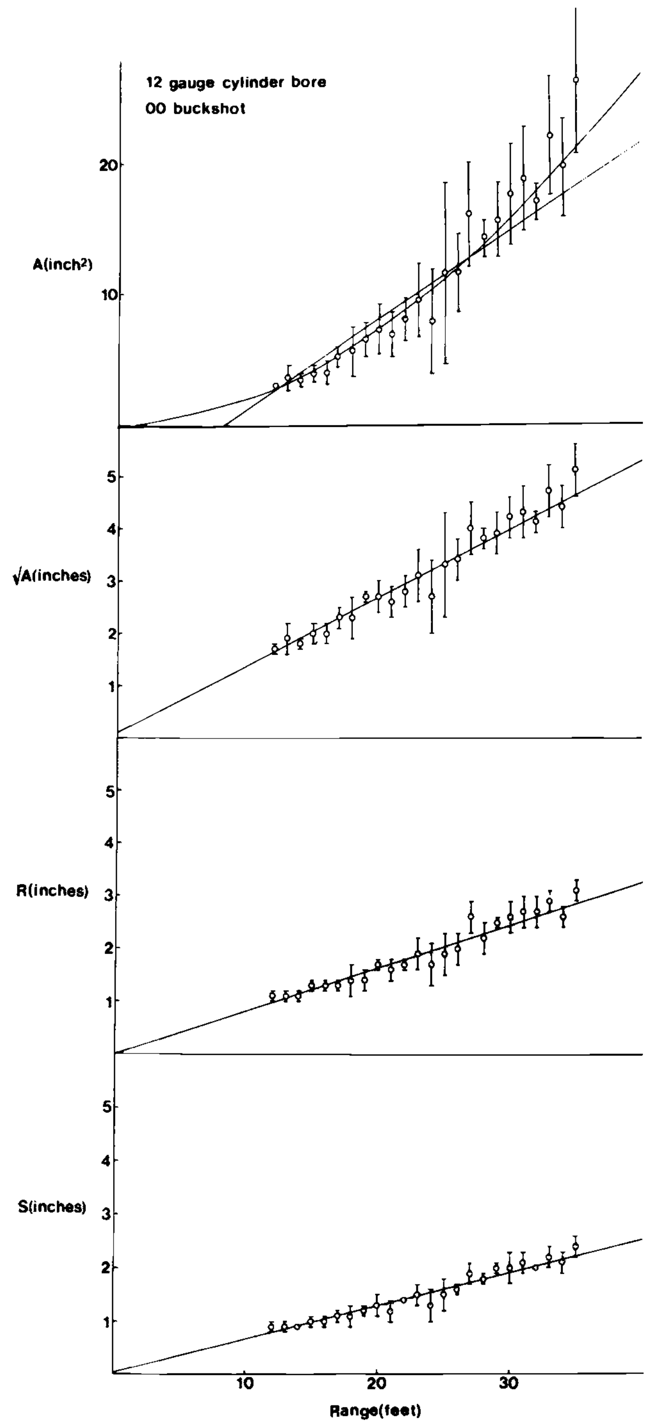

FIG. 2—*Graphs of* A, $\sqrt{A}$, R, *and* S *as function of the range of fire. Error bars represent* ± *one standard deviation. Regression lines shown were obtained by weighted least squares with the weights calculated directly from the standard deviations. 1 in. = 25.4 mm and 1 ft = 0.3 m.*

where

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \tag{10}$$

and t is student's t for the desired confidence level and $n - 2$ degrees of freedom. Heaney and Rowe [5] have discussed the application of this confidence interval expression to the problem of range-of-fire estimation and the effect of increasing the number of test-fired patterns on the confidence interval of $\hat{x}_o$. Strictly speaking, this confidence interval expression is valid only where

$$\frac{t^2 S_e^2}{b^2 \sum_{i=1}^{n} (x_i - \bar{x})^2} \ll 1 \tag{11}$$

When this is not the case, the more exact formulas of Brownlee [3, p. 286] or Draper and Smith [6, pp. 47-51] must be used.

Examination of Table 1 and Fig. 2 shows that the standard deviations of S, R, and A are not constant: they tend to increase with range, although not uniformly. In such a case, a weighted least squares analysis is appropriate. The explicit formulas below are derived from the matrix formulation of weighted least squares given by Draper and Smith [6, pp. 108-115].

For each x_i there is a standard deviation s_i. Each s_i^2 may be written

$$s_i^2 = c_i s^2 \tag{12}$$

The equations for a, b, and r now become

$$a = \frac{1}{\Delta}\left(\sum_{i=1}^{n} \frac{x_i^2}{c_i} \sum_{i=1}^{n} \frac{y_i}{c_i} - \sum_{i=1}^{n} \frac{x_i}{c_i} \sum_{i=1}^{n} \frac{x_i y_i}{c_i} \right) \tag{13}$$

$$b = \frac{1}{\Delta}\left(\sum_{i=1}^{n} \frac{1}{c_i} \sum_{i=1}^{n} \frac{x_i y_i}{c_i} - \sum_{i=1}^{n} \frac{x_i}{c_i} \sum_{i=1}^{n} \frac{y_i}{c_i} \right) \tag{14}$$

$$\Delta = \sum_{i=1}^{n} \frac{1}{c_i} \sum_{i=1}^{n} \frac{x_i^2}{c_i} - \left(\sum_{i=1}^{n} \frac{x_i}{c_i} \right)^2 \tag{15}$$

$$r = \frac{\sum_{i=1}^{n} \frac{1}{c_i} \sum_{i=1}^{n} \frac{x_i y_i}{c_i} - \sum_{i=1}^{n} \frac{x_i}{c_i} \sum_{i=1}^{n} \frac{y_i}{c_i}}{\left\{ \left[\sum_{i=1}^{n} \frac{1}{c_i} \sum_{i=1}^{n} \frac{x_i^2}{c_i} - \left(\sum_{i=1}^{n} \frac{x_i}{c_i} \right)^2 \right] \left[\sum_{i=1}^{n} \frac{1}{c_i} \sum_{i=1}^{n} \frac{y_i^2}{c_i} - \left(\sum_{i=1}^{n} \frac{y_i}{c_i} \right)^2 \right] \right\}^{1/2}} \tag{16}$$

The standard error of estimate may be calculated as before; however, S_e is no longer of direct significance in calculating confidence intervals. The confidence interval for an estimated $\hat{x}_o$ derived from a y_o which is the mean of m y values with standard deviation s_o is

$$\hat{x}_o \pm \frac{ts}{|b|} \left\{ \frac{c_o}{m} + \frac{1}{n} + \frac{(y_o - a - b\bar{x})^2}{b^2 \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{c_i}} \right\}^{1/2} \tag{17}$$

where

$$c_o = s_o^2/s^2 \tag{18}$$

This confidence interval expression is derived under the following constraints:

$$\sum_{i=1}^{n} \frac{1}{c_i} = n \tag{19}$$

and

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} \frac{x_i}{c_i} \tag{20}$$

The defining equation for s^2 now becomes

$$s^2 = \frac{n}{\sum_{i=1}^{n} \frac{1}{s_i^2}} \tag{21}$$

The confidence interval expression is valid only when

$$\frac{t^2 s^2}{b^2 \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{c_i}} \ll 1 \tag{22}$$

When this is not the case, the exact formulas of Brownlee [*3*, p. 316] or Draper and Smith [*6*, pp. 125-126] must be used.

In the usual range-of-fire estimation problem there is only a single questioned pattern, so that $m = 1$. The standard deviation s_o is unknown, but may be estimated from a knowledge of the variation of the s_is with the x_is. If $\hat{s}_o$ is the estimated standard deviation corresponding to y_o, then the confidence interval expression becomes

$$\hat{x}_o \pm \frac{ts}{|b|} \left\{ \hat{c}_o + \frac{1}{n} + \frac{(y_o - a - b\bar{x})^2}{b^2 \sum_{i=1}^{n} (x_i - \bar{x})^2/c_i} \right\}^{1/2} \tag{23}$$

where

$$\hat{s}_o^2 = \hat{c}_o s^2 \tag{24}$$

If the regression line is based on a large number of test-fired rounds so that

$$1/n \simeq 0 \quad \text{and} \quad \frac{(y_o - a - b\bar{x})^2}{b^2 \sum_{i=1}^{n} (x_i - \bar{x})^2/c_i}$$

is small, the confidence interval expression becomes approximately

$$\hat{x}_o \pm \frac{t\hat{s}_o}{|b|} \tag{25}$$

A weighted least squares analysis was used to fit a linear function to the data for S, R, $\sqrt{A}$, and A versus range of fire. In addition to a, b, and r, $t^2 s^2/b^2 \Sigma_{i=1}^{n} [(x_i - \bar{x})^2/c_i]$ was calculated to verify the applicability of the confidence interval expression above. Since s^2, $\bar{x}$, and $1/b^2 \Sigma_{i=1}^{n} [(x_i - \bar{x})^2/c_i]$ are required for confidence interval estimation these were also calculated. The data for all 72 rounds were used in these calculations, rather than the means given in Table 1. Since the same number of replicate shots were fired at each range, either the original data or the means could be used in the calculation of a and b; however, the remaining calculated expressions must be computed from the original data.

The appropriate weighting factors $1/c_i$ may be determined in one of two ways: the experimentally determined standard deviations may be used to calculate the $1/c_i$s [*3,4,6*], or the standard deviations may be fitted to a regression equation in x, from which an estimated standard deviation is obtained for each x_i and used to calculate $1/c_i$ [*3,6*]. The second approach is the more time-consuming; however, in theory it results in an improved value of the standard deviation at each x_i by "pooling" the standard deviations for all the different ranges of fire. This procedure also permits the estimation of the standard deviation for a questioned pellet pattern. The standard deviation data in the present study were fitted to a linear model, since this model permits standard deviations that increase with increasing range. It has been found in many applications that such a model is appropriate [*3*, p. 308]. For purposes of comparison, the weighting factors were determined both from the original standard deviation and from the fitted equation of standard deviation as a function of range.

A weighted least squares analysis was also used to fit the data for A versus range of fire to a quadratic function, namely,

$$y = a + bx + cx^2$$

The regression coefficients a, b, and c were obtained as was the multiple correlation coefficient [*4*, p. 131]. Because confidence intervals for estimates of range would in this case require the solution of quartic equations [*6*, pp. 125–126], the regression analysis was not extended to the calculation of the standard error of estimate or other quantities pertinent to confidence interval calculations. The weighting factors in this case were calculated from the original standard deviations without recourse to a fitting procedure.

Results and Discussion

The means and standard deviations of the dispersal S, radius R, area A, and square root of the area $\sqrt{A}$ for each range are shown in Table 1. These data are shown plotted versus range in Fig. 2. As may be seen, the standard deviations of A increase dramatically with range and are always much larger than the standard deviations of S, R, or $\sqrt{A}$. The area measure of pellet pattern spread is apparently very susceptible to the effects of the erratic flight of so-called "flyers," pellets that impact the target outside the main pattern area. The exclusion of these erratic outliers might improve the shot-to-shot consistency of A; however, such exclusion would be to a large extent subjective and arbitrary, because statistical tests would allow such exclusion only in very rare cases. The exclusion of outliers or flyers could have serious consequences for range estimations based on 00 buckshot patterns, since the exclusion of a single pellet as a flyer represents the omission of 1/9 or 1/12 of the total pattern (depending on the number of pellets in the shot cartridge).

Table 2 presents the results of the regression analysis performed on the data for all 72 test-fired rounds. As may be seen in Table 2, the linear correlation coefficients indicate that a linear function adequately represents the relationship between S, R, and $\sqrt{A}$ and the range of fire, while a quadratic function better represents the relationship between A and the range of fire. The method of weighting does effect the values of the regression coefficients and the linear correlation coefficients, but only to a small degree. Clearly, using the weights calculated directly from the standard deviations results in better linear correlations.

The values of $s^2/b^2 \Sigma_{i=1}^{n} [(x_i - \bar{x})^2/c_i]$ may be used to test the validity of the approximate confidence interval expression (Eqs 17 and 23). Table 3 gives the values of $t^2 s^2/b^2 \Sigma_{i=1}^{n} [(x_i -$

TABLE 2—*Results of weighted linear regression.*[a]

Measure of Pattern Spread	Regression Coefficients a	Regression Coefficients b	Regression Coefficients c	Multiple Correlation Coefficient	Linear Correlation Coefficient r	s^2	$s^2 \Big/ b^2 \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{c_i}$	$\bar{x}$, ft	$1 \Big/ b^2 \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{c_i}$
(a) Weights calculated directly from standard deviations									
Dispersal S (in.)	0.1 in.	0.061 in./ft	...	...	0.982	0.0078 in.^{2}	0.00048	22.7 ft	0.062/in.^{2}
Radius R (in.)	0.0 in.	0.083 in./ft	...	...	0.971	0.013 in.^{2}	0.00073	18.1 ft	0.056/in.^{2}
Area A (in.^{2})	−5.6 in.^{2}	0.68 in.^{2}/ft	...	...	0.942	0.77 in.^{4}	0.0012	14.4 ft	0.0016/in.^{4}
$\sqrt{A}$ (in.)	0.1 in.	0.13 in./ft	...	...	0.966	0.034 in.^{2}	0.00079	17.8 ft	0.023/in.^{2}
(b) Weights calculated from regression analysis of standard deviations									
Dispersal S (in.)	0.0 in.	0.064 in./ft	...	...	0.946	0.019 in.^{2}	0.0014	20.8 ft	0.074/in.^{2}
(standard deviation = 0.1 + 0.0039 × in.; r = 0.371)									
Radius R (in.)	0.0 in.	0.083 in./ft	...	...	0.937	0.033 in.^{2}	0.0016	19.4 ft	0.049/in.^{2}
(standard deviation = 0.0 + 0.0083 × in.; r = 0.485)									
Area A (in.^{2})	−6.5 in.^{2}	0.73 in.^{2}/ft	...	...	0.906	2.3 in.^{4}	0.0023	16.0 ft	0.0010/in.^{4}
(standard deviation = −1.4 + 0.17 × in.^{2}; r = 0.673)									
$\sqrt{A}$ (in.)	0.0 in.	0.13 in./ft	...	...	0.934	0.10 in.^{2}	0.0019	19.5 ft	0.018/in.^{2}
(standard deviation = 0.0 + 0.014 × in.; r = 0.471)									
(c) Fit of area A to a quadratic function									
Area A (in.^{2})	−0.1 in.^{2}	0.043 in.^{2}/ft	0.016 in.^{2}/ft^2	0.975	...	...	...	...	...

[a] 1 in. = 25.4 mm and 1 ft = 0.3 m.

$\bar{x})^2/c_i]$ for the various measures of pellet pattern spread, calculated at the 99.9% confidence level for 70 degrees of freedom (df $= n - 2$). The approximation of Eqs 17 and 23 is generally considered valid for most purposes if

$$t^2s^2/b^2 \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{c_i} < 0.1 \qquad [4, \text{p. } 287]$$

As may be seen from Table 3, Eqs 17 and 23 should be usable for all the measures of pellet pattern spread.

Table 4 illustrates the calculation of an estimated range of fire and its confidence interval using Eq 23 and the data from Table 2. The estimated standard deviations were obtained using the regression equation in Table 2 (b). The confidence interval was calculated for 70 degrees of freedom (df $= n - 2$) at a 99.9% confidence level ($t \simeq 3.4$). This choice is to some degree arbitrary. Examiners may wish to use lower confidence levels, such as 99 or 95%, or an even higher confidence level may be deemed appropriate in some cases.

The following are some general guidelines for applying regression analysis to range-of-fire estimations:

1. Fire all test shots with the suspect weapon and with ammunition from the same batch as that used to fire the questioned pellet patterns.
2. Fire the test shots over as wide as a spread of ranges as is practical so as to minimize $1/b^2 \Sigma_{i=1}^{n} [(x_i - \bar{x})^2/c_i]$. (This makes it more likely that Eqs 17 and 23 will be valid and also will narrow the confidence interval for the estimated range.)
3. Fire several shots at each range. (This allows a determination of the variation of the standard deviations with range.)
4. Having carried out the regression analysis, test the model function being fitted for appropriateness by examining the linear or multiple correlation coefficients.
5. If the model chosen is deemed appropriate, estimate the standard deviation for the questioned pellet pattern and use Eq 23 to calculate the estimated range of fire and its associated confidence interval.

Summary

Three methods of measuring the spread of shotgun pellet patterns were applied to 72 00 buckshot patterns so that we could determine which method was best fitted by a linear func-

TABLE 3—$t^2s^2/b^2 \Sigma_{i=1}^{n} [(x_i - \bar{x})^2/c_i]$ *at 99.9% confidence level* ($t \simeq 3.4$) *for 70 degrees of freedom* (*df* $= n - 2$).

		$t^2s^2/b^2 \Sigma_{i=1}^{n} [(x_i - \bar{x})^2/c_i]$
(a) WEIGHTS CALCULATED DIRECTLY FROM STANDARD DEVIATIONS		
Dispersal	S	0.0056
Radius	R	0.0083
Area	A	0.0144
	$\sqrt{A}$	0.0091
(b) WEIGHTS CALCULATED FROM REGRESSION ANALYSIS OF STANDARD DEVIATIONS		
Dispersal	S	0.016
Radius	R	0.019
Area	A	0.026
	$\sqrt{A}$	0.022

TABLE 4—*Sample calculation for estimated range of fire and its confidence interval.*[a]

Actual Range = 30 ft $\begin{cases} S = 1.8 \text{ in.} \\ R = 2.3 \text{ in.} \\ A = 13.5 \text{ in.}^2 \\ \sqrt{A} = 3.7 \text{ in.} \end{cases}$

Measure of Pattern Spread		$\hat{x}_o = \frac{y_o - a}{b}$, ft	$\hat{s}_o$[b]	$\hat{c}_o = s_o^2/s^2$	$\pm \frac{ts}{\lvert b \rvert} \left\{ \hat{c}_o + \frac{1}{n} + \frac{(y_o - a - b\bar{x})^2}{b^2 \sum_{i=1}^{n} [(x_i - \bar{x})^2/c_i]} \right\}^{1/2}$, ft[c]
Dispersal S	using Table 2(a)	29	0.17 in.	3.6	±9
	using Table 2(b)	28	0.17 in.	1.4	±9
Radius R	using Table 2(a)	28	0.24 in.	4.6	±10
	using Table 2(b)	28	0.25 in.	1.8	±10
Area A	using Table 2(a)	28	3.3 in.^{2}	13.9	±17
	using Table 2(b)	27	3.1 in.^{2}	4.3	±14
$\sqrt{A}$	using Table 2(a)	28	0.43 in.	5.4	±11
	using Table 2(b)	28	0.43 in.	1.8	±11

[a] 1 in. = 25.4 mm and 1 ft = 0.3 m.

[b] Calculated using the regression equations in Table 2(b).

[c] Calculated at the 99.9% confidence level for 70 degrees of freedom (df = $n - 2$) ($t \simeq 3.4$).

tion of the range of fire and which method yielded the smallest confidence interval. A dispersal method S, essentially the same as the "effective shot dispersion" of Mattoo and Nabar [2], a method in which the area A of the smallest circumscribed rectangle that will just enclose the pellet pattern is calculated, and an overlay method for determining the radius R of the smallest circumscribed circle that will just enclose the pellet pattern were all tested. Of these three, the dispersal S showed the best fit to a linear function of the range and area A the worst. Use of the square root of A resulted in a significant improvement in the fit. The area A was also found to show the highest shot-to-shot variation and gave the widest confidence interval for range-of-fire estimates. The dispersal S gave the smallest confidence interval for range-of-fire estimates; however, the differences in the confidence intervals for $\sqrt{A}$, S, and R were small.

Acknowledgment

We would like to acknowledge the statistical advice provided by Dr. W. D. Foster of the Armed Forces Institute of Pathology. We would also like to thank Captain James Burnett for the use of the overlay in Fig. 1.

References

[*1*] Mattoo, B. N. and Nabar, B. S., "Evaluation of Effective Shot Dispersion in Buckshot Patterns," *Journal of Forensic Sciences*, Vol. 14, No. 2, April 1969, pp. 263-269.

[*2*] DiMaio, V. J. M. and Petty, C. S., "The Effectiveness of the Sawed-off Shotgun," *The Forensic Science Gazette*, Vol. 9, No. 2, April-June 1978, pp. 5-6.

[*3*] Brownlee, K. A., *Statistical Theory and Methodology in Science and Engineering*, Wiley, New York, 1961.

[*4*] Bevington, P. R., *Data Reduction and Error Analysis for the Physical Sciences*, McGraw-Hill, New York, 1969.

[*5*] Heaney, K. D. and Rowe, W. F., "The Application of Linear Regression to Range-of-Fire Estimates Based on the Spread of Shotgun Pellet Patterns," *Journal of Forensic Sciences*, Vol. 28, No. 2, April 1983, pp. 433-436.

[*6*] Draper, N. R. and Smith, H., *Applied Regression Analysis*, 2nd ed., Wiley, New York, 1981.

Address request for reprints or additional information to
Walter F. Rowe, Ph.D.
Department of Forensic Sciences
The George Washington University
Washington, DC 20052